Definition 8.1

The *direct product* of groups $G_1, \dots, G_n$ is a group $G_1 \times G_2 \times \dots \times G_n$ defined as follows:

- Elements: n-tuples $(g_1, g_2, \dots, g_n)$ where $g_i \in G_i$.
- Group operation:

$$(g_1, g_2, \dots, g_n) \cdot (h_1, h_2, \dots, h_n) = (g_1 h_1, g_2 h_2, \dots, g_n h_n)$$

- The identity element: $(e_1, e_2, \dots, e_n)$ where e_i is the identity element in G_i.
- Inverses: $(g_1, g_2, \dots, g_n)^{-1} = (g_1^{-1}, g_2^{-1}, \dots, g_n^{-1})$.

Note. We have:

$$|G_1 \times G_2 \times \dots \times G_n| = |G_1| \cdot |G_2| \cdot \dots \cdot |G_n|$$

Theorem 8.2

The group $G_1 \times \ldots \times G_n$ is abelian of and only if each of the groups G_i is abelian.

Recall:

- The *least common multiple* of integers $n_1, n_2, \ldots, n_k \geq 1$ is the smallest positive integer, denoted by $\mathrm{lcm}(n_1, \ldots, n_k)$, which is divisible by each of these numbers.
- If $m > 0$ is an integer divisible by $n_1, \ldots, n_k$ then m is divisible by $\mathrm{lcm}(n_1, \ldots, n_k)$.

Theorem 8.3

For $i = 1, \ldots, n$ let $a_i \in G_i$, and let $(a_1, \ldots, a_n) \in G_1 \times \ldots \times G_n$. Then

$$|(a_1, \ldots, a_n)| = \mathrm{lcm}(|a_1|, \ldots, |a_n|)$$