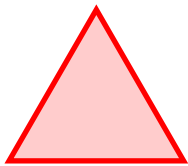
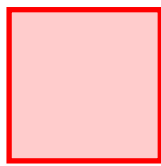
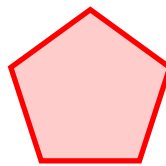
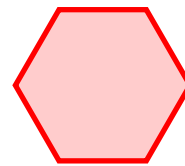


Regular polygons P_n with n sides:

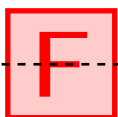
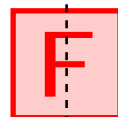
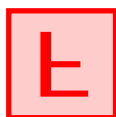
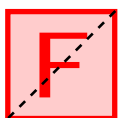
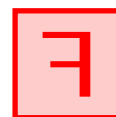
 P_3  P_4  P_5  P_6

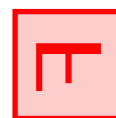
Symmetries of P_4 :


 \xrightarrow{I}

 $\xrightarrow{R_{90}}$

 $\xrightarrow{R_{180}}$

 $\xrightarrow{R_{270}}$

 \xrightarrow{H}

 \xrightarrow{V}

 \xrightarrow{D}

 $\xrightarrow{D'}$


Composition of symmetries:

Composition table of symmetries of a square:

\circ	I	R_{90}	R_{180}	R_{270}	H	V	D	D'
I	I	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_{90}	R_{90}	R_{180}	R_{270}	I	D'	D	H	V
R_{180}	R_{180}	R_{270}	I	R_{90}	V	H	D'	D
R_{270}	R_{270}	I	R_{90}	R_{180}	D	D'	V	H
H	H	D	V	D'	I	R_{180}	R_{90}	R_{270}
V	V	D'	H	D	R_{180}	I	R_{270}	R_{90}
D	D	H	D'	V	R_{270}	R_{90}	I	R_{180}
D'	D'	V	D	H	R_{90}	R_{270}	R_{180}	I

For $n \geq 3$ the dihedral group D_n is defined as follows:

- **Elements of D_n :** symmetries of the regular polygon with n sides.
- **Group operation:** Composition of symmetries (e.g. $V \circ D = R_{270}$).
- **The identity element:** The identity symmetry I .

Definition 5.1

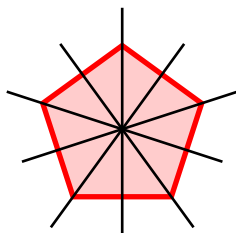
The *order* of a group G is the number of elements of G . It is denoted by $|G|$. If G has infinitely many elements, we write $|G| = \infty$.

Theorem 5.2

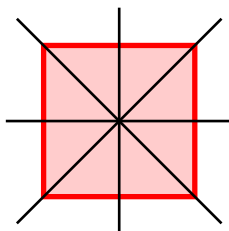
For $n \geq 3$ we have $|D_n| = 2n$.

Note. The dihedral group D_n consists of the following elements:

- 1) n rotations by the angles of $k \cdot \frac{360}{n}$ degrees for $k = 0, \dots, n - 1$. For $k = 0$ this gives the rotation by 0 degrees, i.e. the identity symmetry.
- 2) n reflections with respect to different symmetry axes. If n is odd, there is one symmetry axis for each vertex of the polygon P_n :



If n is even, there are $\frac{n}{2}$ symmetry axes passing through pairs of opposite vertices and $\frac{n}{2}$ symmetry axes crossing opposite sides of P_n :



Definition 5.3

Let G be a group, and let $S \subseteq G$ be a subset of G . We say that the set S *generates* G if every element of G can be obtained as a product of some elements of S and inverses of elements of S .