Definition 23.1

Let R be an integral domain, and let $a, b \in R$. We say that a *divides* b if $b = ac$ for some $c \in R$. We then write: $a \mid b$.

Theorem 23.2

If R is an integral domain and $a, b \in R$ are non-zero elements then $a \sim b$ and only if $a \mid b$ and $b \mid a$.

Theorem 23.3

If R is an integral domain and $a \in R$ is a prime element then a is irreducible.

Theorem 23.4

If R is a UFD and $a \in R$ then a is an irreducible element if and only if a is a prime element.

Theorem 23.5

An integral domain R is a UFD if and only if the following conditions are satisfied:

1) Every non-zero, non-unit element of R is a product of irreducible elements.

2) Every irreducible element in R is a prime element.