Definition 13.1

Let G be a group and $H \subseteq G$ a subgroup. For $a \in G$ the *left coset of H in G containing a* is the subset of G given by

$$aH = \{ah \mid h \in H\}$$

Similarly, the *right coset of H in G containing a* is the subset

$$Ha = \{ha \mid h \in H\}$$

Example. Consider the group D_4:

$\circ$	I	R_{90}	R_{180}	R_{270}	H	V	D	D'
I	I	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_{90}	R_{90}	R_{180}	R_{270}	I	D'	D	H	V
R_{180}	R_{180}	R_{270}	I	R_{90}	V	H	D'	D
R_{270}	R_{270}	I	R_{90}	R_{180}	D	D'	V	H
H	H	D	V	D'	I	R_{180}	R_{90}	R_{270}
V	V	D'	H	D	R_{180}	I	R_{270}	R_{90}
D	D	H	D'	V	R_{270}	R_{90}	I	R_{180}
D'	D'	V	D	H	R_{90}	R_{270}	R_{180}	I

Theorem 13.2

Let G be a group, $H \subseteq G$ a subgroup, and let $a, b \in G$. Then:

1) $a \in aH$.

2) either $aH = bH$ or $aH \cap bH = \varnothing$.

3) $aH = bH$ if and only if $a^{-1}b \in H$.

4) $|aH| = |H|$, where $|aH|$ denotes the number of elements in aH.

Analogous properties hold for right cosets.

Definition 13.3

For a group G and a subgroup $H \subseteq G$ by G/H we denote the set of left cosets of H in G and by $H \backslash G$ we denote the set of right cosets.

Theorem 13.4

If G is a group and $H \subseteq G$ is a subgroup, then $|G/H| = |H \backslash G|$.

Definition 13.5

If G is a group and $H \subseteq G$ is a subgroup then the *index* of H, denoted $[G : H]$, is the number of left cosets of H in G (or, equivalently, the number of right cosets):

$$[G : H] = |G/H| = |H \backslash G|$$

Theorem 13.6 (Lagrange Theorem)

If G is a finite group and $H \subseteq G$ is a subgroup then

$$|G| = [G : H] \cdot |H|$$

Corollary 13.7

If G is a finite group and $H \subseteq G$ is a subgroup then the order of H divides the order of G.

Corollary 13.8

If G is a finite group and $a \in G$ then the order $|a|$ of a divides the order of G.