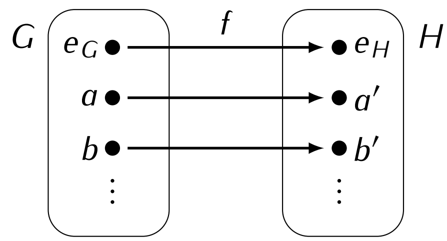
**Definition 12.1**

An *isomorphism of groups* is a group homomorphism which is both onto and 1-1.




**Theorem 12.2**

If $g\colon G \to H$ is an isomorphism then the inverse function $f^{-1}\colon H \to G$ is also an isomorphism.

**Theorem 12.3**

A homorphism of groups $f\colon G \to H$ is an isomorphism if and only if $\mathrm{Im}(f) = H$ and $\mathrm{Ker}(f) = \{e\}$.

**Definition 12.4**

We say the group $G$ is *isomorphic* to a group $H$ if there exists an isomorphism $f\colon G \to H$. Then we write $G \cong H$.

**Theorem 12.5**

Isomorphism of groups is an equivalence relation:

**1)** For any group $G$ we have $G \cong G$.

**2)** If $G, H$ are groups such that $G \cong H$ then $H \cong G$.

**3)** If $G, H, K$ are groups such that $G \cong H$ and $H \cong K$, then $G \cong K$.

**Theorem 12.6 (Cayley's Theorem)**

Let $G$ be a finite group of order $n$. Then $G$ is isomorphic to a subgroup of the symmetric group $S_n$.

**Definition 12.7**

An *automophism* of a group $G$ is an isomorphism $f\colon G \to G$.

**Definition 12.8**

Let $G$ be a group. The *group of automorphisms* of $G$ is the group $\mathrm{Aut}(G)$ whose elements are automorphisms of $G$ and the group operation is given by composition of automorphisms.