For full credit explain your reasoning, showing all relevant work.

Exercise 1.

Let G, H_1, H_2 be groups and let $f_1 \colon G \to H_1$, $f_2 \colon G \to H_2$ be homomorphisms. Show that the function $f \colon G \to H_1 \times H_2$ given by $f(a) = (f_1(a), f_2(a))$ is a homomorphism of groups.

Exercise 2.

a) Describe all possible homomorphisms $f \colon \mathbb{Q} \to \mathbb{Z}$, where $\mathbb{Q}$ is the group of rational numbers and $\mathbb{Z}$ is the group of integers (both taken with addition as the group operation). Justify your answer.

Exercise 3. Let G be a group. Define a new group G' which has the same elements as G, but such that the multiplication in G' is given by

$$\underbrace{a \odot b}_{\text{multiplication in } G'} = \underbrace{b \cdot a}_{\text{multiplication in } G}$$

Show that the group G' is isomorphic to G.

PRACTICE PROBLEMS

Exercises below are for practice only - do not turn them in for grading.

Practice Exercise 1.

a) Describe all possible group homomorphisms $f\colon \mathbb{Z}_6 \to \mathbb{Z}_{15}$.

b) For each of these homomorphisms f compute $\mathrm{Ker}(f)$.

Practice Exercise 2. Let G be a group. Show that the function $f\colon G \to G$ defined by $f(a) = a^2$ is a homomorphism if and only if G is an abelian group.

Practice Exercise 3. Let G be a group. Consider a homomorphism $f\colon G \to G \times G$ given by $f(a) = (a, a)$. Show that the group G is abelian if and only if $\mathrm{Im}(f)$ is a normal subgroup of $G \times G$.

Practice Exercise 4. Show that the group $\mathbb{Z}_4$ is not isomorphic to the group $\mathbb{Z}_2 \times \mathbb{Z}_2$.