For full credit explain your reasoning, showing all relevant work.

Exercise 1. Consider a permutation $\alpha \in S_5$ which is a product of two cycles:

$$\alpha = (1, 2, 3, 5) \cdot (4, 1, 3)$$

a) Write α in the matrix notation.

b) Write α as a product of disjoint cycles.

c) Write α^{-1} in the matrix notation.

d) Compute the order of α.

Exercise 2. a) Compute all possible orders of elements in S_7.

b) Recall that A_n denotes the subgroup of S_n consisting of all even permutations. Compute all possible orders of elements in A_7.

Exercise 3. Compute the number of elements of order 12 in S_7.

PRACTICE PROBLEMS

Exercises below are for practice only - do not turn them in for grading.

Practice Exercise 1. Let $\beta \in S_6$ be a permutation given as the following product of cycles

$$\beta = (1,4,5,6,2) \cdot (2,3,4,5) \cdot (1,3,6) \cdot (2,3,5)$$

Write β^{10} as a product of disjoint cycles.

Practice Exercise 2. Compute the number of elements of order 6 in S_7.

Practice Exercise 3. Find an element of order 30 in the group A_{12}.

Practice Exercise 4. Let H be a subgroup of S_n. Show that if there is $\alpha \in H$ which is an odd permutation, then exactly half elements of H are odd and half are even.