For full credit explain your reasoning, showing all relevant work.

Exercise 1. Let $G = \langle a \rangle$ be a cyclic group of order 20 generated by an element a.
a) What are all possible generators of G? That is, what are elements a^k such that $G = \langle a^k \rangle$?

b) Let H be a subgroup of G of order 10. What are all possible generators of H? That is, what are elements a^k such that $H = \langle a^k \rangle$?

Exercise 2. a) Let G be a cyclic group which has exactly 3 different subgroups: the whole group G, the trivial subgroup $\{e\}$ and a subgroup H of order 11. What is the order of G? Justify your answer.

b) Let G be a cyclic group which has as the only subgroups the whole group G, and groups of orders 1, 2, 4, 7, 14. What is the order of G? Justify your answer.

Exercise 3. Let $\mathbb{Q}$ be the group of rational numbers with addition. Show that $\mathbb{Q}$ is not a cyclic group.

PRACTICE PROBLEMS

Exercises below are for practice only - do not turn them in for grading.

Practice Exercise 1. Find all elements of order 9 in the group $\mathbb{Z}_{900}$. Explain how you know that these are all elements of order 9 in this group.

Practice Exercise 2. Let $\mathbb{Q}^+$ denote the group of positive rational numbers with multiplication. Show that $\mathbb{Q}^+$ is not a cyclic group.

Practice Exercise 3. For each integer $n \geq 1$ give an example of a group G which has exactly n different subgroups (including the whole group and the trivial subgroup).

Practice Exercise 4. Show that if G is an infinite group then it has infinitely many different subgroups.