For full credit explain your reasoning, showing all relevant work.

Exercise 1. Let G be a group and let $a, b, c \in G$ be elements such that $|a| = 6$ and $|b| = 7$. Express the element

$$(a^4c^{-2}b^4)^{-1}$$

without using any negative exponents.

Exercise 2. Let D_3 be the dihedral group of symmetries of an equilateral triangle.

a) Let $A \subseteq D_3$ be a subset consisting of third powers of elements of D_3:

$$A = \{g^3 \in D_3 \mid g \in D_3\}$$

Is A a subgroup of D_3? Justify your answer.

b) Let $B \subseteq D_3$ be a subset consisting of squares of elements of D_3:

$$B = \{g^2 \in D_3 \mid g \in D_3\}$$

Is B a subgroup of D_3? Justify your answer.

Hint: See section 5 of the lecture notes for a description of elements of groups D_n.

Exercise 3. A *proper subgroup* of a group G is a subgroup $H \subseteq G$ which is not equal to the whole group G.

Let G be a group and let $H_1, H_2 \subseteq G$ be two proper subgroups of G. Show that there exists an element $g \in G$ such that $g \notin H_1$ and $g \notin H_2$.

PRACTICE PROBLEMS

Exercises below are for practice only - do not turn them in for grading.

Practice Exercise 1. Let G be a group and let A_1, A_2 be two subgroups of G. Define a subset $A_1A_2 \subseteq G$ by

$$A_1A_2 = \{a_1a_2 \mid a_1 \in A_1, a_2 \in A_2\}$$

a) Consider the dihedral group D_4:

$\circ$	I	R_{90}	R_{180}	R_{270}	H	V	D	D'
I	I	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_{90}	R_{90}	R_{180}	R_{270}	I	D'	D	H	V
R_{180}	R_{180}	R_{270}	I	R_{90}	V	H	D'	D
R_{270}	R_{270}	I	R_{90}	R_{180}	D	D'	V	H
H	H	D	V	D'	I	R_{180}	R_{90}	R_{270}
V	V	D'	H	D	R_{180}	I	R_{270}	R_{90}
D	D	H	D'	V	R_{270}	R_{90}	I	R_{180}
D'	D'	V	D	H	R_{90}	R_{270}	R_{180}	I

Take the subgroups of D_4 given by $A_1 = \{I, D\}$ and $A_2 = \{I, H\}$. Which elements are in the set A_1A_2? Which elements are in the set A_2A_1? Is the set A_1A_2a subgroup of D_4?

b) Show that if A_1, A_2 are subgroups of a group G such that $A_1A_2 = A_2A_1$ then A_1A_2 is a subgroup of G.

Practice Exercise 2. Assume that G is a group which has exactly one element $a \in G$ of order 2. Show that for any $g \in G$ we have $ag = ga$.

Practice Exercise 3. Recall that the dihedral group D_6 is the group of symmetries of a regular polygon with 6 sides. Let $H \subseteq D_6$ be a subset consisting of squares of elements of D_6:

$$H = \{g^2 \mid g \in D_6\}$$

Show that H is a subgroup of D_6.

Hint: See section 5 of the lecture notes for a description of elements of groups D_n.

Practice Exercise 4. Assume that in a group G there exists an element a of order pk where p is a prime number. Let $b \in G$ be an element such that $b^p = a$. Show that $|b| = p^2k$.