For full credit explain your reasoning, showing all relevant work.

Exercise 1. Compute multiplication tables for groups with 4 different elements e, a, b, c (where e is the identity element) in each of the following cases:

1) $a \cdot a = b \cdot b = c \cdot c = e$
2) $a \cdot a = a$,
3) $a \cdot a = b$

Note. All other possible multiplication tables can be obtained from these by relabeling elements. For example, the multiplication table in the case where $a \cdot a = c$ is the same as in the case $a \cdot a = b$, but with b and c swapped.

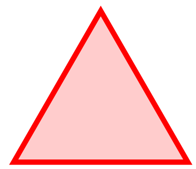
Exercise 2. Recall that if G is a group and $a \in G$, then $|a|$ denotes the order of a. Show that for any $a, b \in G$ we have $|a| = |bab^{-1}|$.

Exercise 3. Let G be a finite group such that $|G|$ is even. Show that the number of elements of order 2 in G is odd. In particular, since 0 is an even number, G contains at least one element of order 2.

PRACTICE PROBLEMS

Exercises below are for practice only - do not turn them in for grading.

Practice Exercise 1. a) Describe all elements of the group D_3, the dihedral group of symmetries an equilateral triangle.



b) Compute the multiplication table of the group D_3.

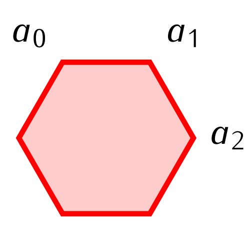
c) Is D_3 an abelian group? Why or why not?

d) Calculate the order of each element of D_3.

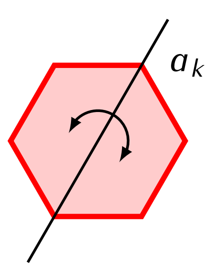
Practice Exercise 2. Let G be a group and let $g \in G$. Show that $|g| = |g^{-1}|$.

Practice Exercise 3. Let G be a finite group and let $n > 2$. Show that the number of elements of G of order n is even.

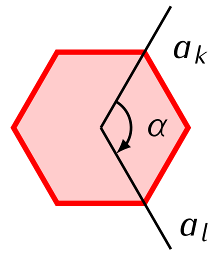
Practice Exercise 4. Let D_n be the dihedral group of symmetries of a regular polygon with n vertices. Label vertices of the polygon by $a_0, a_1, \dots, a_{n-1}$:



Let $V_k \in D_n$ denote the symmetry that reflects the polygon about the axis passing through the vertex a_k:



Show that the composition $V_l \circ V_k$ is the rotation by the angle 2α where α is the angle from the vertex k to the vertex l:



In other words, $V_l \circ V_k$ is the rotation that takes the vertex a_0 to the vertex a_r where $r = 2(l - k) \mod n$.