For full credit explain your reasoning, showing all relevant work.

Exercise 1. Compute values of the following expressions in the group $U(12)$:

  a) $5 \cdot 7$
  b) 11^{-1}
  c) $5 \cdot 11^{-1}$

Exercise 2. Let H denote be set of real numbers greater than 1. For $a, b \in H$ define

$$a \odot b := ab - a - b + 2$$

Show that H taken with this multiplication is a group. What is the identity element in this group? For $a \in H$ what is a^{-1}?

Exercise 3. Let G be a group such that $a^2 = e$ for all $a \in G$ (where e is the identity element in G). Show that G is an abelian group.

Exercises below are for practice only – do not turn them in for grading.

Practice Exercise 1. Compute values of the following expressions in the group $\mathbb{Z}_{12}$:

  a) $10 + 5$
  b) $4 + 8$
  c) $4 - 8$

Practice Exercise 2. Find all elements of the group $U(20)$ and the inverse for each of them.

Practice Exercise 3. Show that for any $n > 2$ there are at least two different elements $a, b \in U(n)$ such that $a^2 = b^2 = 1$.

Practice Exercise 4. Let G be a finite group. Show that the number of elements $a \in G$ such that $a^3 = e$ is odd.