Sets

In general sets will be denoted by capital letters: $A, B, C, \ldots$

Frequently used sets:

$\varnothing =$ the empty set (i.e. the set that contains no elements)

$\mathbb{N} = \{0, 1, 2, \ldots\}$ the set of natural numbers

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ the set of integers

$\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$ the set of positive integers

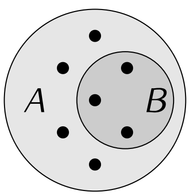
$\mathbb{Q} =$ the set of rational numbers

$\mathbb{R} =$ the set of real numbers

We will write $x \in A$ to denote that x is an element of the set A and $y \notin A$ to indicate that y is not an element of A. For example, $5 \in \mathbb{Z}$, $\frac{1}{3} \notin \mathbb{Z}$.

Subsets

A set B is a *subset* of a set A if every element of B is in A. In such case we write $B \subseteq A$.



A set B is a *proper subset* of A if $B \subseteq A$ and $B \neq A$.

Example. Some subsets of $\mathbb{Z}$:

- $A = \{n \in \mathbb{Z} \mid n > 10\}$ – the set of integers greater than 10.
- $B = \{n \in \mathbb{Z} \mid n = 2k \text{ for some } k \in \mathbb{Z}\}$ – the set of even integers.

Operations on sets

• The *union* of sets A and B is the set $A \cup B$ that consists of all elements that belong to either A or B (or both):

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

• The *intersection* of sets A and B is the set $A \cap B$ that consists of all elements that belong to both A and B:

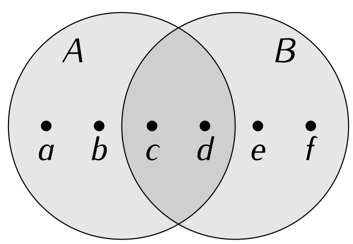
$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

• The *difference* of sets A and B is the set $A \setminus B$ that consists of all elements that belong to A but not to B:

$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$

Example. If $A = \{a, b, c, d\}$, $B = \{c, d, e, f\}$ then:

$$\begin{aligned} A \cup B &= \{a, b, c, d, e, f\} \\ A \cap B &= \{c, d\} \\ A \setminus B &= \{a, b\} \end{aligned}$$



Note. We say that sets A and B are *disjoint sets* if $A \cap B = \varnothing$.

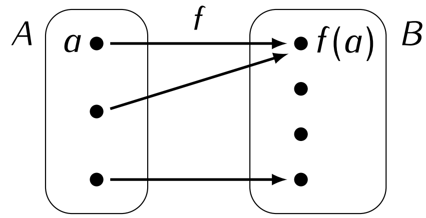
• The *Cartesian product* of sets A, B is the set consisting of all ordered pairs of elements of A and B:

$$A \times B = \{(a, b) \mid a \in A, \quad b \in B\}$$

Example. If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ then

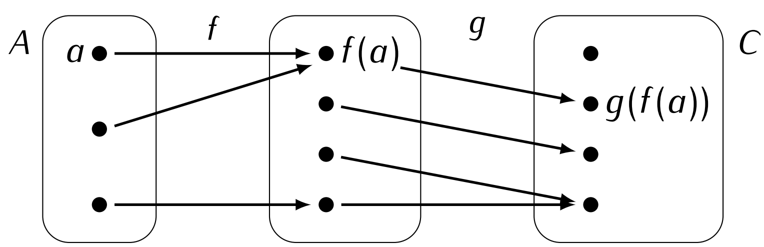
$$A \times B = \{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$$

<u>**Functions**</u>



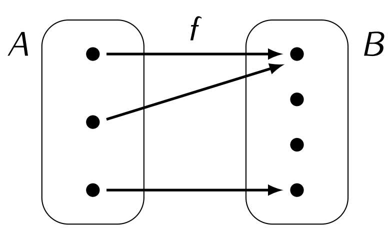
$$f\colon A \to B$$

Composition of functions:

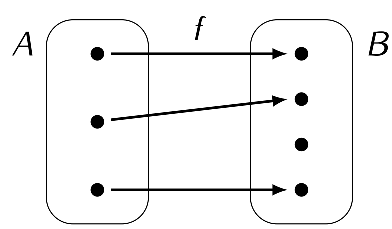


$$g \circ f\colon A \to C$$

• A function $f\colon A \to B$ is *1-1* if $f(a) = f(a')$ only if $a = a'$.

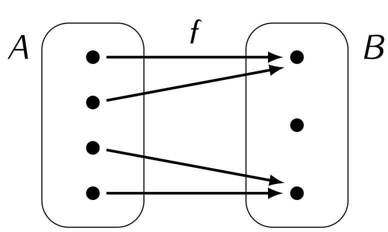


not 1-1

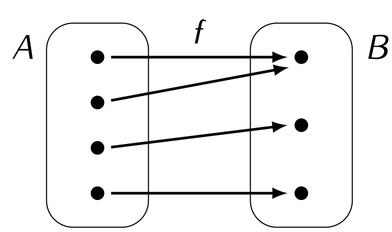


1-1

• A function $f\colon A \to B$ is *onto* if for every $b \in B$ there is $a \in A$ such that $f(a) = b$

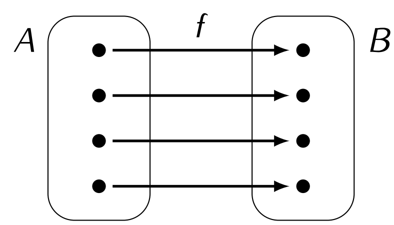


not onto



onto

- A function $f\colon A \to B$ is a *bijection* if f is both 1-1 and onto.



bijection

Note. If $f\colon A \to B$ is a bijection then the inverse function $f^{-1}\colon B \to A$ exists and it is also a bijection. Then for every $a \in A$ and $b \in B$ we have:

$$f^{-1}(f(a)) = a \quad \text{and} \quad f(f^{-1}(b)) = b$$

Cardinality

We will denote by $|A|$ the cardinality of the set A. For a finite set, this is the number of elements of A.

Note. If A, B are sets, then $|A| = |B|$ if and only if there exists a bijection $f\colon A \to B$.